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Holographic $T\bar{T}$ deformed entanglement entropy in dS_3/CFT_2

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Abstract

In this paper, based on the $T\bar{T}$ deformed version of dS_3/CFT_2 correspondence, we calculate the pseudoentropy for an entangling surface consisting of two antipodal points on a sphere and find it is exactly dual to the complex geodesic in the bulk.

1 Introduction

The study of quantum gravity in de Sitter space has generated much interest in recent years, particularly due to its potential relevance for inflationary cosmology and cosmic acceleration. One promising method to comprehend de Sitter space is through the dS/CFT correspondence [1]. It is a conjectured equivalence between a gravitational theory in de Sitter space and a conformal field theory residing on its boundary. The dS/CFT correspondence is a generalization of the well-known AdS/CFT correspondence [2,3], which has been extensively studied in string theory and provided numerous insights into extracting the nature of quantum gravity from its dual CFT. However, the dS/CFT correspondence is not as well understood as the AdS/CFT correspondence, as there are only limited explicit examples of CFTs that are dual to de Sitter spacetime. Recently, a remarkable and explicit example has been constructed for the $\mathrm{dS}_3/\mathrm{CFT}_2$ correspondence [4,5], where the dual CFT resides on the past/future boundary of de Sitter spacetime.

Starting with the three-dimensional de Sitter spacetime, we take a static compact slice Σ_t of constant time. Clearly, each Σ_t has a Riemannian metric γ and a second fundamental form K. In the canonical formalism for gravity, the quantum state residing on Σ_t can be described by the Hartle-Hawking wavefunction $\Psi_{\mathrm{dS}}[\gamma]$. Following [4,5], and neglecting the contributions of bulk matter fields, we could obtain a calculable example of $\mathrm{dS}_3/\mathrm{CFT}_2$ correspondence described by

$$\Psi_{\mathrm{dS}}[\gamma] = Z_{\mathrm{CFT}}[\gamma]\,, \quad t \to \infty \tag{1.1}$$

where Z_{CFT} is the partition function of the dual CFT_2 living on Σ_∞.

In this paper, we aim to further explore the scenario described above. Typically, it is not necessary to confine the slice Σ_t at the future infinity, which leads to a natural extension of the $\mathrm{dS}_3/\mathrm{CFT}_2$ correspondence:

$$\Psi_{\mathrm{dS}}[\gamma] = Z_{\mathrm{QFT}}[\gamma]\,, \tag{1.2}$$

where Z_{QFT} is the partition function of the dual quantum field theory(QFT) living on a finite-volume slice Σ_t. The dual QFT could be defined as a $2d$ CFT deformed by the $T\bar{T}$ operator [6–8] that generates a trajectory in the space of field theory,

$$\frac{\partial}{\partial\lambda}\log Z(\lambda) = -2\pi\int_\Sigma d^2x\sqrt{\gamma}\left\langle T\bar{T}\right\rangle_\lambda . \tag{1.3}$$

At the first order of the deformation parameter λ, the deformed theory, perturbatively, could be written as

$$\log Z(\lambda) = \log Z(\lambda=0) - 2\pi\lambda\int_\Sigma d^2x\sqrt{\gamma}\left\langle T\bar{T}\right\rangle_{\lambda=0} + \mathcal{O}(\lambda^2), \tag{1.4}$$

where $\left\langle T\bar{T}\right\rangle_{\lambda=0}$ is defined by the stress tensor of the undeformed theory as $\left\langle T\bar{T}\right\rangle_{\lambda=0} \equiv \left\langle T\bar{T}\right\rangle = \frac{1}{8}\left[\left\langle T^{ab}\right\rangle\left\langle T_{ab}\right\rangle - \left\langle T^a_a\right\rangle^2\right]$. In recent years, the $T\bar{T}$ deformation has been widely studied [9–41], due to its integrability and its applications in holography. Our proposal is a natural extension of the cutoff-AdS/$T\bar{T}$-deformed-CFT correspondence [42] to de Sitter spacetime. Additionally, the $T\bar{T}$-deformed version (1.2) of $\mathrm{dS}_3/\mathrm{CFT}_2$ is remarkably coincident with the Cauchy slice holography [43, 44], where time serves as the emergent direction. The $T\bar{T}$-deformed version (1.2) of $\mathrm{dS}_3/\mathrm{CFT}_2$ is illustrated in Figure 1.

It is clear that the $T\bar{T}$ deformation is irrelevant in the renormalization group sense. This implies that the $T\bar{T}$ deformation leads no consequence in IR but does affect UV physics. Among the various deformable

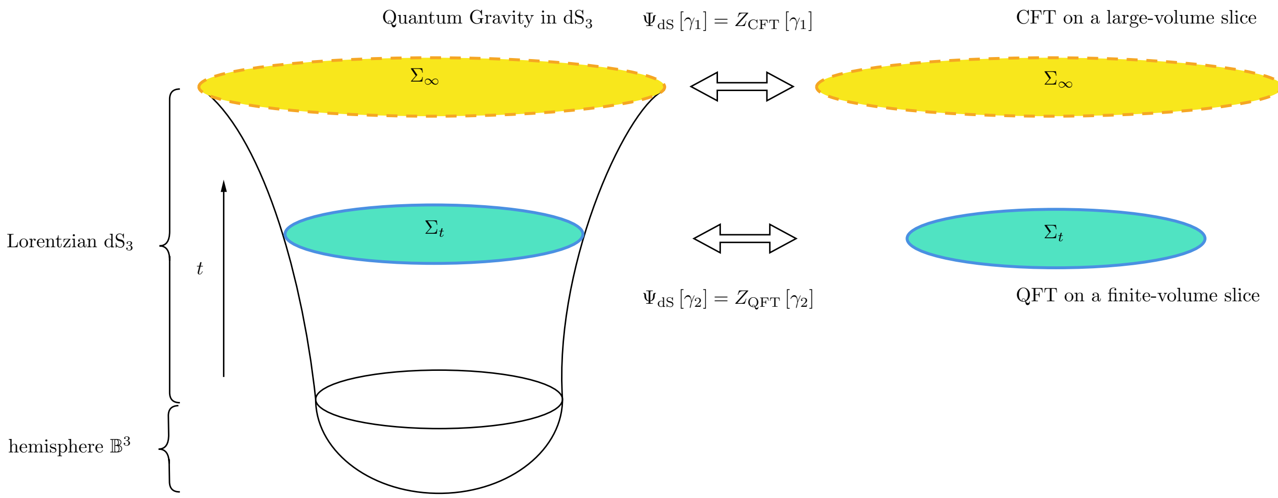

Figure 1: The $T\bar{T}$-deformed version of the dS_3/CFT_2 correspondence, with the Lorentzian time t in the global coordinates of the dS_3 spacetime.

physical quantities in the UV region, a particularly important one is the entanglement entropy. In the dS/CFT correspondence, the dual CFTs turn to be non-unitary [1, 4, 5, 45]. To characterize the degrees of freedom in a non-unitary CFT, complex-valued entanglement entropies, namely pseudoentropy [39–41, 46–54], are needed. In other words, the pseudoentropy can be viewed as a well-defined entanglement entropy in the $T\bar{T}$-deformed version of the dS_3/CFT_2 correspondence. It is then interesting to explore how the holographic entanglement entropy [55, 56] behaves in the $T\bar{T}$-deformed version of the dS_3/CFT_2. In this paper, our main goal is to calculate the entanglement entropy in the $T\bar{T}$-deformed field theory and compare it with geodesics in the dS_3 bulk.

In section 2, we give a brief review of the $T\bar{T}$ deformed version of dS/CFT. In section 3, we calculate the pseudoentropy for an entangling surface consisting of two antipodal points on a sphere $\mathbb{S}^2$, and we find that the entanglement entropy does perfectly match the length of the complex geodesic connecting these antipodal points in dS_3.

2 Wheeler-DeWitt equation and $T\bar{T}$ flow

We first briefly review the $T\bar{T}$ deformed version of dS/CFT in this section. In the canonical formalism of three-dimensional pure gravity, the Hartle-Hawking wavefunction $\widetilde{\Psi}[\gamma]$ should obey the Wheeler-DeWitt equation

$$\mathcal{H}\widetilde{\Psi}[\gamma] = \left\{\frac{16\pi G_N}{\sqrt{\gamma}}\left(\Pi^{ab}\Pi_{ab} - \Pi^a_a\Pi^b_b\right) - \frac{\sqrt{\gamma}}{16\pi G_N}\left(\mathcal{R}[\gamma] - 2\Lambda\right)\right\}\widetilde{\Psi}[\gamma] = 0, \tag{2.5}$$

where $\mathcal{R}[\gamma]$ is the Ricci scalar, $\Lambda = \ell_{\rm dS}^{-2}$ is the cosmological constant of the de Sitter spacetime, and Π^{ab} is the momentum conjugate to the metric γ_{ab}:

$$\Pi^{ab} = -\mathrm{i}\frac{\delta}{\delta\gamma_{ab}} = \frac{\sqrt{\gamma}}{16\pi G_N}\left(K^{ab} - K^c_c\gamma^{ab}\right). \tag{2.6}$$

The standard quasilocal stress tensor can be defined as:

$$T^{ab} = -\frac{2}{\sqrt{\gamma}}\frac{\delta}{\delta\gamma_{ab}} = -\frac{2\mathrm{i}}{\sqrt{\gamma}}\Pi^{ab}, \tag{2.7}$$

which coincides with the field-theoretic definition. To require the finiteness of the quasilocal stress tensor at the future infinity Σ_∞, one needs to perform a canonical transformation [44, 57],

$$\widetilde{\Psi} = \exp\left(-\frac{\mathrm{i}}{8\pi G_N \ell_{\mathrm{dS}}}\int_\Sigma d^2x\sqrt{\gamma}\right)\Psi, \tag{2.8}$$

which leads a shift on the momentum

$$\Pi^{ab} \to \Pi^{ab} + \frac{\sqrt{\gamma}}{16\pi G_N \ell_{\mathrm{dS}}}\gamma^{ab}. \tag{2.9}$$

Therefore, the Wheeler-DeWitt equation could be rewritten as

$$\left\{-\frac{2}{\sqrt{\gamma}}\Pi^a_a + \frac{16\pi G_N \ell_{\mathrm{dS}}}{\det\gamma}\left(\Pi^{ab}\Pi_{ab} - \Pi^a_a\Pi^b_b\right) - \frac{\ell_{\mathrm{dS}}}{16\pi G_N}\mathcal{R}\left[\gamma\right]\right\}\Psi\left[\gamma\right] = 0. \tag{2.10}$$

By using the quasilocal stress tensor, the equation is simply

$$T^a_a = \frac{\mathrm{i}\,\ell_{\mathrm{dS}}}{16\pi G_N}\mathcal{R}\left[\gamma\right] + \mathrm{i}\,4\pi G_N \ell_{\mathrm{dS}}\left(T^{ab}T_{ab} - T^a_a T^b_b\right). \tag{2.11}$$

On the other hand, in the $T\bar{T}$ deformed field theory, when the deformation parameter λ is small, one can rewrite eqn.(1.3) as

$$\log Z_{\mathrm{QFT}} = \log Z_{\mathrm{CFT}} - 2\pi\lambda\int_\Sigma d^2x\sqrt{\gamma}\left\langle T\bar{T}\right\rangle. \tag{2.12}$$

By the definition of the trace of the stress tensor

$$T^a_a = -2\frac{\gamma_{ab}}{\sqrt{\gamma}}\frac{\delta}{\delta\gamma_{ab}}\log Z, \tag{2.13}$$

and the famous Weyl anomaly

$$\left(T^a_a\right)_{\mathrm{CFT}} = -\frac{c}{24\pi}\mathcal{R}\left[\gamma\right], \tag{2.14}$$

the trace flow equation for deformed theory is

$$\left\langle T^a_a\right\rangle = -\frac{c}{24\pi}\mathcal{R}\left[\gamma\right] - \frac{\pi\lambda}{2}\left(\left\langle T^{ab}\right\rangle\left\langle T_{ab}\right\rangle - \left\langle T^a_a\right\rangle\left\langle T^b_b\right\rangle\right). \tag{2.15}$$

Relating to eqn.(2.11), we immediately find the identifications between field-theoretic quantities and gravitational quantities

$$c = -\mathrm{i}\frac{3\ell_{\mathrm{dS}}}{2G_N} = -\mathrm{i}\,c_{\mathrm{dS}}, \quad \lambda = -\mathrm{i}\,8G_N\ell_{\mathrm{dS}} = -\mathrm{i}\,\lambda_{\mathrm{dS}}, \tag{2.16}$$

where the Brown-Henneaux central charge [58] turns to be imaginary-valued in the de Sitter context [45], and the deformation parameter is also imaginary-valued. Furthermore, the momentum constraint for the Hartle-Hawking wavefunction,

$$\mathcal{D}^a\Psi\left[\gamma\right] = -2\left(\nabla_b\Pi^{ab}\right)\Psi\left[\gamma\right] = 0, \tag{2.17}$$

can be easily interpreted as the conversation law of the stress tensor in field theory

$$\nabla_b\left\langle T^{ab}\right\rangle = 0. \tag{2.18}$$

Even though the dual field theory is non-unitary, the dynamical inner product $\langle\Psi|\Psi\rangle$ is Hermitian and positive-semidefinite, which indicates that we still have the bulk unitarity [43, 44].

3 Holographic Entanglement Entropy

First, we briefly introduce the pseudoentropy. Dividing the total system into two subsystems A and B, the pseudoentropy is defined by the von Neumann entropy,

$$S_A = -\mathrm{Tr}\left[\tau_A \log \tau_A\right], \tag{3.19}$$

of the reduced transition matrix

$$\tau_A = \mathrm{Tr}_B \left[\frac{|\psi\rangle \langle\varphi|}{\langle\varphi \mid \psi\rangle}\right], \tag{3.20}$$

where $|\psi\rangle$ and $|\varphi\rangle$ are two different quantum states in the total Hilbert space that is factorized as $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. For a generic QFT living on a curved surface Σ, the pseudoentropy could be captured by the replica method [59, 60] in path integral formalism. Denoting the manifold corresponding to $\langle\varphi \mid \psi\rangle$ as Σ and the manifold corresponding to $\mathrm{Tr}_A\left(\tau_A\right)^n$ as Σ_n, the pseudoentropy for the subsystem A reads

$$S_A = \lim_{n\to 1} \frac{1}{1-n} \log\left[\frac{Z_{\Sigma_n}}{\left(Z_\Sigma\right)^n}\right], \tag{3.21}$$

where Z_Σ is the path integral over the manifold Σ and S_A can be regarded as a well-defined entanglement entropy in the dS_3 context.

To capture the pseudoentropy, we first need to calculate the partition function for a field theory. Specifically, one saddle solution for the Hartle-Hawking wavefunction $\Psi_{\mathrm{dS}}\left[\gamma\right]$ is the Euclidean sphere $\mathbb{S}^2$, where the corresponding metric of de Sitter spacetime is given by

$$ds^2 = \ell_{\mathrm{dS}}^2 \left(-dt^2 + \cosh^2 t\, d\Omega_2^2\right), \tag{3.22}$$

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta\, d\phi^2$ is the metric of a $2d$ unit sphere, and the spacelike boundary at time t is a Euclidean sphere $\mathbb{S}^2$ with the radius $r = \sqrt{\frac{\lambda_{\mathrm{dS}}\, c_{\mathrm{dS}}}{12}} \cosh t$. In this section, we will calculate the pseudoentropy of the $T\bar{T}$ deformed field theory living on a sphere $\mathbb{S}^2$ with a radius r. To be precise, we focus on the case that an entangling surface consists of two antipodal points on this sphere. Following [10], for a field theory living on a sphere with the metric

$$ds^2 = r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{3.23}$$

the stress tensor takes the form

$$T_{ab} = \alpha \gamma_{ab}, \tag{3.24}$$

where α could be determined by the trace flow equation,

$$\alpha = \frac{1}{\pi\lambda}\left(1 - \sqrt{1 + \frac{\lambda c}{12 r^2}}\right) = \frac{\mathrm{i}}{\pi\lambda_{\mathrm{dS}}}\left(1 - \sqrt{1 - \frac{\lambda_{\mathrm{dS}}\, c_{\mathrm{dS}}}{12 r^2}}\right). \tag{3.25}$$

Noticing that $2\partial_r \gamma_{ab} = r\gamma_{ab}$, one obtains the equation for the partition function

$$\frac{d\log Z_{\mathrm{QFT}}}{dr} = -\frac{1}{r}\int_\Sigma d^2x \sqrt{\gamma}\, T_a^a = \mathrm{i}\frac{8}{\lambda_{\mathrm{dS}}}\left(\sqrt{r^2 - \frac{\lambda_{\mathrm{dS}}\, c_{\mathrm{dS}}}{12}} - r\right), \tag{3.26}$$

and the partition function thus reads

$$\log Z_{\mathrm{QFT}} = \mathrm{i}\frac{4}{\lambda_{\mathrm{dS}}}\left[r\left(\sqrt{r^2 - \frac{\lambda_{\mathrm{dS}}\, c_{\mathrm{dS}}}{12}} - r\right) - \frac{\lambda_{\mathrm{dS}}\, c_{\mathrm{dS}}}{12}\tanh^{-1}\left(\frac{r}{\sqrt{r^2 - \lambda_{\mathrm{dS}}\, c_{\mathrm{dS}}/12}}\right)\right]. \tag{3.27}$$

It is worthy to note that, since $r/\sqrt{r^2-\lambda_{\rm dS}\,c_{\rm dS}/12}>1$, the function $\tanh^{-1}\left(r/\sqrt{r^2-\lambda_{\rm dS}\,c_{\rm dS}/12}\right)$ is indeed complex-valued. Focusing on the principal branch of the inverse hyperbolic function, one then obtains

$$\log Z_{\rm QFT}={\rm i}\frac{4}{\lambda_{\rm dS}}\left[r\left(\sqrt{r^2-\frac{\lambda_{\rm dS}\,c_{\rm dS}}{12}}-r\right)-\frac{\lambda_{\rm dS}\,c_{\rm dS}}{12}\coth^{-1}\left(\frac{r}{\sqrt{r^2-\lambda_{\rm dS}\,c_{\rm dS}/12}}\right)\right]+\frac{\pi c_{\rm dS}}{6}. \tag{3.28}$$

The real part $\frac{\pi c_{\rm dS}}{6}$ is consistent with the result in [5]:

$$\left|Z_{\rm CFT}\right|^2=\exp\left(\frac{\pi c_{\rm dS}}{3}\right) \tag{3.29}$$

since the deformation parameter is imaginary-valued and the $T\bar{T}$ deformation only affects the imaginary part of $\log Z$.

Utilizing the replica method introduced in [10], we could obtain the pseudoentropy S_A for an entangling surface of two antipodal points on a sphere $\mathbb{S}^2$:

$$S=\left(1-\frac{r}{2}\frac{d}{dr}\right)\log Z_{\rm QFT}=-{\rm i}\,\frac{c_{\rm dS}}{3}\coth^{-1}\left(\frac{r}{\sqrt{r^2-\lambda_{\rm dS}\,c_{\rm dS}/12}}\right)+\frac{\pi c_{\rm dS}}{6}. \tag{3.30}$$

Substituting $r=\sqrt{\frac{\lambda_{\rm dS}\,c_{\rm dS}}{12}}\cosh t$, the entanglement entropy of two antipodal points on a $\mathbb{S}^2$ is thus given by

$$S_A=\frac{c_{\rm dS}}{6}\pi-{\rm i}\,\frac{c_{\rm dS}}{3}t. \tag{3.31}$$

On the other hand, in the dS_3 bulk, the geodesic distance between two points at the same time $(t,\theta_i=0,0)$ and $(t,\theta_f=\pi,0)$ is given by

$$D\left(\theta_i,\theta_f\right)=\ell_{\rm dS}\mathrm{arcos}\left[1-2\cosh^2 t\sin^2\left(\frac{\theta_f-\theta_i}{2}\right)\right]=\ell_{\rm dS}\pi-2\,{\rm i}\,\ell_{\rm dS}t. \tag{3.32}$$

Using the identifications eqn.(2.16), the RT formula gives

$$S_A=\frac{D\left(\theta_i,\theta_f\right)}{4G_N}=\frac{c_{\rm dS}}{6}\pi-{\rm i}\,\frac{c_{\rm dS}}{3}t, \tag{3.33}$$

which exactly is equal to the entanglement entropy eqn. (3.31).

Therefore, as promised, we verified that, for a static finite volume slice $\mathbb{S}^2$ in dS_3, the pseudoentropy for an entangling surface consisting of two antipodal points is precisely equal to the complex geodesic in the dS_3 bulk.

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