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Bulk Physics, Algebras and All That
Part Two: Black Hole Information Problem

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Abstract: This is a discussion on some modern aspects of AdS/CFT, with particular focus on entanglement entropy, bulk reconstruction, and operator algebraic aspects of bulk and boundary subregions. Keep in mind that these are a very informal collection of things that I found interesting, and are in no way meant to be a review or formal introduction whatsoever.

In Part Two, we will discuss the black hole information paradox in an elementary fashion. We will not focus on AdS/CFT aspects too much, but instead more or less provide introduction for the BHIP in general. Note that this is no longer a Three Part series and is an N-part series, with large N limit.

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Contents

1 Introduction

Part One was based on bulk reconstruction and subregion duality in AdS/CFT. We will now talk about the black hole information problem (BHIP) and the features that AdS/CFT has that closely provide resolutions to the BHIP. The reader surely must be familiar with the paradox regarding information, black holes and the famous Hawking-Preskill-Thorne wager. Of course, black holes are not easy things to make sense of, and have been the center of attention in hep-th for quite some time now (see fig 1). AdS/CFT has brought an entirely new perspective on this problem, and the bulk-boundary physics in this theory has shed light on this important problem.

Figure 1. Black holes when someone tries to understand them.

While the traditional phrase to this entire scheme is the "black hole information *paradox*", we are obliged to call it a problem to (slightly) exaggerate the look of this landscape as a problem. Black holes are, as it must be clear to anyone having picked up some hep-th papers on the arXiv, definitely not an easy thing to make sense of. A good deal of information theory is involved when dealing with the BHIP in general, and most importantly in the AdS/CFT counterpart to the BHIP, there are many reconstruction aspects that are involved. For instance, one could ask if the interior of the black hole is something that can be made sense of by instead making measurements to the CFT side. The purpose of these notes is to address these things in a nice pedagogical way. The structure of these notes is as follows.

In section 2, we will quickly recap some basics of the BHIP, and the setup used will be the usual Schwarzschild geometry. We will recap Hawking's arguments on the evaporation of black holes in semiclassical gravity, and the Page curve. We will talk about the generalized second law as formulated by Bekenstein [1], and some arguments therein. We will review complementarity [2, 3], and motivate AMPS [4] and firewalls via the monogamy paradox [5]. We do not, however, review in detail firewalls and fuzzballs and only allude to some arguments around them.

In section 3, we will discuss AdS/CFT aspects of the BHIP, particularly focusing on the Raju-Papadodimas proposal on black hole interior reconstruction [5–7]. We will discuss coupling an AdS black hole theory to a bath in 2D dilaton theory and motivate quantum islands [8]. We will then discuss an ER=EPR paradox with the TFD as argued by Marolf and Wall [9]. Finally, we will discuss briefly some aspects of ER=EPR and (non-)traversable wormholes [10, 11]. For a lighter introduction to BHIP, see Aayush's excellent notes [12].

2 Black Hole Information Problem in a Nutshell

Classical black holes are great. One can do nice things with classical black holes, like work with gravitational waves in mergers, or do lensing, and basically most of the astrophysical works. The No Hair theorem shows that the after evaporation, the only numbers required to describe black holes are M, Q and L, which in itself causes some suspicion. Hawking showed that assuming the null energy condition,

$$T_{\mu\nu}X^{\mu}X^{\nu} \geq 0 \,, \tag{2.1}$$

classical black holes always have an increasing horizon area. Following Bekenstein and Hawking's argument that the entropy of a black hole is proportional to the area of the horizon $S_{BH} \sim A$, it must, therefore, also be that the entropy of a black hole is always increasing. This seems to be kind of nice. However, classical black holes do not evaporate, which was shown to be a semiclassical issue.

But then, in semiclassical physics, would the evaporation and therefore decreasing horizon area imply a violation of the second law of thermodynamics? This seems

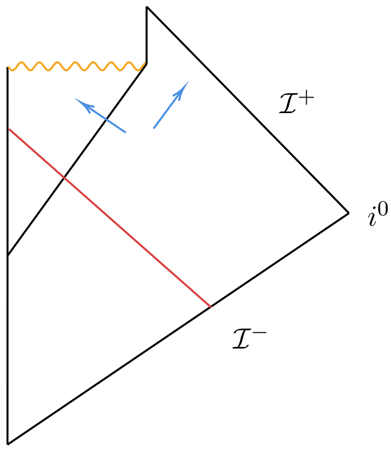

Figure 2. An evaporating black hole. The red line denotes an infalling shell of matter, whereas the blue arrows indicate negative and positive fluxes of particles falling into and escaping the horizon and escaping to $\mathcal{I}^+$ respectively.

so. However, Bekenstein argued that the correct quantity to work with in this situation is the generalized entropy S_{gen},

$$S_{gen} = \frac{\text{Area of Horizon}}{4G_N} + S_{ext} \ . \tag{2.2}$$

The generalized second law of thermodynamics would then concern this quantity's monotonicity properties. One should, therefore, expect that as the horizon area decreases, the exterior entanglement entropy compensates for this. However, there are subtleties with how this von Neumann entropy functions, as we shall discuss. We will first start by doing some quick analysis in the free scalar field situation across the horizon, and the origin of Hawking radiation.

Let us sit in the M^{D+1} spacetime with the metric

$$ds^2 = -\mathrm{d}U\mathrm{d}V + \delta_{\mu\nu}dx^\mu dx^\nu \ . \tag{2.3}$$

One also has to expect deviations to this form of the metric, but this is not very essential here. One could pick a semiclassical state $|\Psi\rangle$ and compute the two correlator $\langle\phi(x_1)\phi(x_2)\rangle_\Psi$. The general form looks like

$$\frac{\Gamma(D-1)}{2^D\pi^{D/2}\Gamma\left(\frac{D}{2}\right)}\frac{1}{\gamma^{\frac{D-1}{2}}}\left(1+\mathrm{O}(\gamma)\right) \ , \tag{2.4}$$

where γ is the geodesic distance between x_1 and x_2. The coordinates are nice to work with to emphasise on the importance of correlations across null surfaces. The Schwarzschild setup can be written in terms of the tortoise coordinate r^*, which blows up to negative infinity at the horizon $r = H$, by requiring

$$\frac{1}{f(r)} \equiv 1 - \frac{\mathcal{M}}{r^2} = \frac{dr^*}{dr} \ ,$$

where

$$\mathcal{M} = \frac{8GM\pi^{1-\frac{D}{2}}\Gamma\left(\frac{D}{2}\right)}{D-1} \ .$$

Then, the metric becomes

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega_{D-1}^2 \ . \tag{2.5}$$

We now want to work with small scale correlations around the horizon. Taking a scalar field description, one has the Klein-Gordon solutions, denoted by $\xi^{\text{in}}(\omega, l, r^*)$ and $\xi^{\text{out}}(\omega, l, r^*)$. Here, l and ω are the angular quantum numbers and the frequency respectively. $\xi^{\text{in}}(\omega, l, r^*)$ in the r approaching H from outside limit, denoted by $r \to H^+$ looks like

$$\xi^{\text{in}}(\omega, l, r^*) = \chi_{\omega,l} e^{-i\omega r^*} \ , \tag{2.6}$$

whereas $\xi^{\text{out}}(\omega, l, r^*)$ in the $r \to H^-$ limit looks like

$$\xi^{\text{out}}(\omega, l, r^*) = e^{i\omega r^*} + \chi'_{\omega,l} e^{-i\omega r^*} \ . \tag{2.7}$$

The factors $\chi_{\omega,l}$ and $\chi'_{\omega,l}$ are not needed in this analysis, but can be computed nonetheless. Then, one can write a field in terms of these something like

$$\phi = \sum \int d\omega \ \left[\mathcal{A}\xi^{\text{in}}(\omega, l, r^*) + \mathcal{B}\xi^{\text{out}(\omega,l,r^*)}\right] e^{-i\omega t} Y(\Omega) + \text{ hermitian conjugates} \ . \tag{2.8}$$

In the Kruskal coordinates, we have

$$U = \frac{-1}{\kappa} e^{\kappa(r^*-t)} \ , \quad \text{and} \tag{2.9}$$

$$V = \frac{1}{\kappa} e^{\kappa(r^*+t)} \ . \tag{2.10}$$

With precise calculations of $\mathfrak{a}$ and $\tilde{\mathfrak{a}}$, and the smearing functions associated to $a_{\omega,l}$ and $a^\dagger_{\omega,l}$, the reader is directed to 2012.05770 and 1910.02992. For now, it is only relevant that $\mathfrak{a} = a_{\omega,l}$ and $\tilde{\mathfrak{a}} = \tilde{a}_{\omega,l}$ with a normalized commutator. The two-point function for this then is

$$\langle a_{\omega,l} a^\dagger_{\omega,l} \rangle_\Psi = \frac{1}{1 - e^{-\beta\omega}} \ , \tag{2.11}$$

where β is the inverse temperature. This implies that there is a flux at $\mathcal{I}^+$.

2.1 Page Curve

Now, here is our situation: we know from Bekenstein and Hawking's famous formula,

$$S = \frac{A}{4} \ , \tag{2.12}$$

that the area of the horizon is proportional to the entropy of the black hole. Since Hawking radiation exists, the positive flux to $\mathcal{I}^+$ tells us that the black hole evaporates, leading to a decreasing area of the horizon. So, at the least, we expect that the generalized second law holds [1], and (2.2) is monotonically increasing. Here is where we encounter the *Page curve* dilemma.

The natural information theoretic interpretation of the Page curve [5, 13] is that one could take a full system $\mathcal{H}$, and pick a small subsystem S. If one takes the ratio of the von Neumann entropy in terms of the ratio of S to $\mathcal{H}$, one would find that it obeys a very characteristic nature: it increases upto a certain point, at which $S = \frac{1}{2}\mathcal{H}$. Then, since the subsystem S becomes the larger system and $S' = \mathcal{H} - S$ becomes smaller, the curve decreases steadily. This is the Page curve for $\mathcal{H}$.

What we expect of black holes can be seen from usual aspects of von Neumann entropy and states. The von Neumann entropy

$$S_{vN} = -\mathrm{Tr}(\rho \log \rho) \tag{2.13}$$

has the property that for pure states $|\psi\rangle$, it vanishes. So we would expect that the density matrix ρ looks like $|\psi\rangle\langle\psi|$, for which the eigenvalues look like

$$\rho = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} .$$

An additional property is that the maximum value of S_{vN} is given by the logarithm of the dimensionality of the Hilbert space $\mathcal{H}$.

Entropy bounds, however, also play a role here; one expects that the Bekenstein bound (which is a certain limit of the Bousso bound)

$$S \leq 2\pi E \mathcal{R} , \tag{2.14}$$

where E is the energy and $\mathcal{R}$ is the radius of the smallest area sphere packing the system. Then, denoting by S_{rad} the entropy of Hawking radiation, we expect $S_{\text{rad}} = 0$ for the pure state. We would expect that this, eventually goes back to zero, i.e. a pure state. However, Hawking's famous calculation showed that this is not the case; instead, it increases, and saturates the Bekenstein bound without ever reaching zero again. See fig. 3.

Now this is bad. This tells us that in some sense, unitarity is being lost. One can again see the usual expectation by noting that the dimension of the Hilbert spaces of Hawking radiation and the evaporating black hole are supposed to compensate for one another.

The more explicit calculation of the number of these particles generating this flux is due to Hawking. Note that from the first law of black hole thermodynamics,

$$dM = \frac{\kappa}{\pi} dA + \Phi dQ + \Omega dL . \tag{2.15}$$

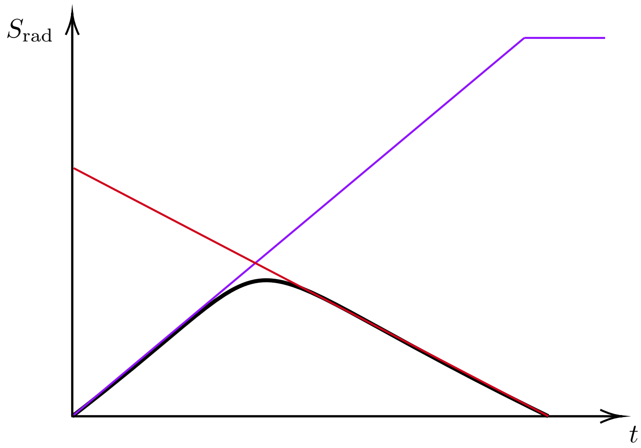

Figure 3. The Page curve. The red line is the thermodynamic entropy of the black hole, which is monotonically decreasing as the black hole evaporates. The thick black line is the Page curve, whereas Hawking's calculation is the violet line.

Then, the number of these particles, $N_{j\omega lmp}$ is [14]

$$\Gamma_{j\omega lmp}\left[\exp\left(2\pi\kappa^{-1}\left(\omega - e\Phi - m\Omega\right) \mp 1\right)\right]^{-1} \ , \tag{2.16}$$

where the subscripts j, l, m and p denote species, spherical harmonic, angular quantum number and helicity. The minus and plus signs are for bosons or fermions respectively.

2.2 Generalized Second Law

Let us now concern ourselves with a little more on what the role of the GSL really is. Firstly, it is clear that this is a highly non-trivial law, since one has an evaporating black hole without a proper construct of the variation of S_{gen}. A nice modern approach to this is in terms of "holographic screens", where one starts by picking suitable marginally trapped surfaces inside the black hole and construct a holographic screen of such surfaces. In semiclassical gravity, one takes quantum marginally trapped surfaces, i.e. surfaces for which the quantum expansion is negative along one null congruence and zero along the null congruence orthogonal to it. One can define $\mathcal{F}_\lambda(a)$, a one-parameter family of functions that can be used along a null congruence parametrized by the affine λ and a along the spacelike codimension-2 surface I. This satisfies

$$\partial_\lambda \mathcal{F}_\lambda(y) \geq 0 \ .$$

Then, the quantum expansion is defined as

$$\Theta_k(\mathcal{F};\ a) = \frac{4G_N}{\sqrt{\eta}}\frac{\delta S_{gen}(I_\mathcal{F})}{\delta \mathcal{F}_a} \ , \tag{2.17}$$

with the interpretation that similar to the classical expansion, one is making infinitesimal deformations to the surface I along the null congruence to measure the variation of the generalized entropy. Here, η is the induced metric, and we assume that the surface is a compact surface (non-compactness is typically troublesome to work with and plays a key role in singularity theorems). k is the outgoing null congruence, whereas l will denote the ingoing null congruence. Associated to this would be a pair of future and past congruences, so on an all we have $k^{\pm}$ and $l^{\pm}$. The Bousso bound states that the most entropy that can pass through I is bounded by the area of it, assuming the null energy condition (so that the domains of dependence of the lightsheet and the surface are closed and equal):

$$S(I) \leq n\frac{A(I)}{4G_N} , \tag{2.18}$$

where n is the number of lightsheets. In this way, one expects that for (at least) marginally trapped surfaces, this entropy bound is *at most* saturated. The Bekenstein bound can be seen to be a slightly weaker form of this bound. A lot of work has been done for evaporating black holes and such entropy bounds, and in particular the holographic screen approach by Bousso and Engelhardt shows that the GSL is preserved by taking semiclassical corrections to the holographic screen (dubbed the Q-screen).

2.3 Complementarity

In regards to the full nature of physics in the black hole information paradox, Susskind, Thorlacius and Uglum [2, 3] postulated three points dealing with (1) what to expect of quantum field theory for an evaporating black hole, (2) the semiclassical approximation of the field theory and (3) the dimensionality of the subspace giving the black hole a description. These are fully expanded as follows:

1. **Postulate 1.** *An evaporating black hole can be described by usual QFT.* One could make this more precise by saying that there exists an S-matrix describing infalling matter and Hawking radiation:

   $$S\,(\text{infalling matter} \mid \text{Hawking radiation})\ .$$

   In the face of this, there is a nice way of making sense of $\mathcal{I}^+$ in a holographic sense in the asymptotically flat setting. Among many things, in a noncompact situation, like Minkowski spacetime, one has some very fascinating observations, thanks to algebraic QFT. One such result is the split property, which states that one can define a collar region ε around a bounded region $\mathcal{U} \subset M$ on a noncompact Cauchy slice Σ, and for this there exists a type I factor $\mathfrak{R}$ so that the full Hilbert space factorizes like

   $$\mathcal{H} = \mathcal{H}_N \otimes \mathcal{H}_{\bar{N}}\ . \tag{2.19}$$

However, there exists a very interesting property of gravitational QFTs, called *holography of information*, due to which information about $\mathcal{U}\cup\varepsilon$ is *also* available on the boundary of the Cauchy slice $\partial\Sigma$. This is clearly in opposition to the assertion of the split property, which is that the state in $\mathcal{U}\cup\varepsilon$ can be prepared individually to that of the complement. These aspects also appear in the AdS/CFT discussion of the BHIP.

2. **Postulate 2.** *Physics to a good deal is semiclassical understood outside the stretched horizon.*

3. **Postulate 3.** *The dimensionality of the subspace of states describing this black hole is related to the Bekenstein bound:*

$$\text{Dim}\left(\mathcal{H}_{BH(M)}\right) = e^{S_{\mathbf{Bek}}(M)} . \tag{2.20}$$

2.4 Monogamy and Firewalls

One could argue that there is a fourth postulate which largely encompasses the other three postulates of complementarity:

4. **Postulate 4.** *For an infalling observer, the horizon of the evaporating black hole should be natural.*

The argument of Almheiri, Marolf, Polchinski and Sully (AMPS) [4] is that this postulate, along with a bit of the others is somewhat dangerously naive. One expects that there is a bad counter-example of this in the form of "firewalls". Before going there, we will quickly revisit a monogamy paradox. (Monogamy is, after all, a good and ethical thing.)

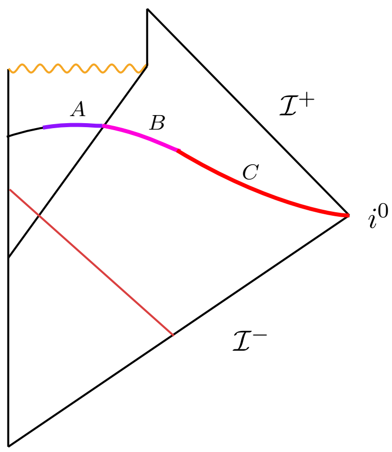

Figure 4. A suitable Cauchy slice, for which there are three sections of interest: A, which lies just inside the horizon, B, which lies just outside the horizon, and C, which extends to the boundary of the Cauchy slice $\partial\Sigma$ at i^0.

One could follow this discussion intently, but at a point realise the meaning of this entire problem for old black holes. Take a black hole geometry so that one can find a "good" Cauchy slice Σ to i^0, so that it is cut into three sections: a region A that lies just inside the horizon, B which lies just outside, and C, which lies at $\partial\Sigma$; the idea being that the slice cuts the interior and the exterior of the black hole and the Hawking radiation. Now, we already know from the discussion above that across the horizon, modes are entangled, so

$$A \longleftrightarrow B \, .$$

Ok. Now, for an old black hole, the near horizon modes are entangled with Hawking radiation extending to $\mathcal{I}^+$. However, this now implies that

$$B \longleftrightarrow C \, ,$$

which goes badly with the monogamy of entanglement. Another way of arriving at this discrepancy is to assume that the Hilbert space of the full theory factorizes like [5]

$$\mathcal{H} \to \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C \, . \tag{2.21}$$

Clearly, this violates strong subadditivity entanglement entropy. One way of resolving this anti-monogamy problem is to go against one of the complementarity postulates. What AMPS presented in a paper titled "*Complementarity or Firewalls?*" is to replace the horizon with a firewall (see fig. 5). Of course, this is against the equivalence principle and complementarity in entirety. So, one could argue that perhaps in a loophole-ish way that after all, one has to *measure* the Hawking radiation first. In Susskind's argument, the two famous protagonists Alice and Bob are made to do these measurements in the *cloning* setup, where Alice jumps in with a qubit, with Bob just outside the horizon, and Bob waits for the information to come out as Hawking radiation, and jumps in. The non-firewall situation in the entanglement picture looks something like this: initially, we only have maximal entanglement between $A \longleftrightarrow B$, and Alice sits in B. In later times, we would have $A \longleftrightarrow C$, and one could get a conservation of entanglement, as Susskind coined it. What the firewall situation does is simplify the paradox by quite a bit – by simply saying that in the cloning setup, by the time Bob can jump in following Alice in early pre-Page time, Bob waits till about at or after the Page time, but he is destroyed at the firewall. Or, if Alice herself waits till the Page time, she herself is destroyed at the firewall. Essentially, one could motivate firewalls simply computationally, by saying that the correlator $\langle T_{\mu\nu} \rangle$ diverges at the firewall.

Susskind then goes on to argue that the firewall situation is not as "neat" as it seems. For instance, when Alice jumps in, what she observes is an *apparent horizon*, which changes the location of the actual horizon. Eventually, Bob jumping in would *also* change the location of the horizon, and seemingly it complicates the firewall

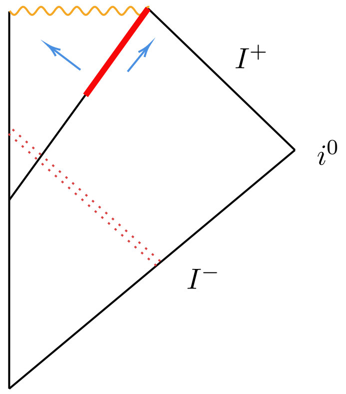

Figure 5. A firewall (thick red line) in a young black hole, with the Hawking radiation identified.

solution. With the scrambling time vs Page time debate, there are many arguments to each side. However, even just the post-Page time situation looks rather complicated. If one views the firewall as an extension of the singularity, and taking into account of the changing horizon, the cloning setup would clearly give a different result without enough satisfaction from the firewalls picture (see fig. 6).

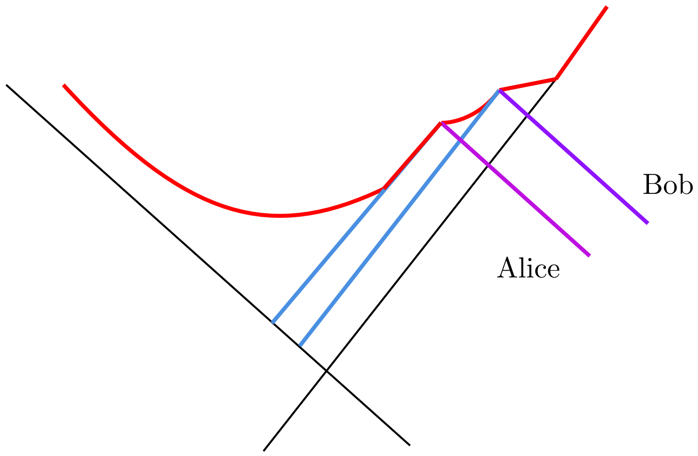

Figure 6. Adapted from Susskind's paper. The firewall, extended from the singularity, and the apparent horizons (thick blue lines) are identified with changes from infalling Alice and Bob.

So, the fuzzballs and firewalls proposals contain some pros and cons:

1. **Firewalls:** Looks good with the monogamy paradox – strong subadditivity of entropy is restored! Goes well with the AMPS, a little worrisome with complementarity. But not preferred, following Susskind's arguments.

2. **Fuzzballs:** Looks good with SUGRA and monogamy paradox, since one no longer has the multiple entanglement partners. But seemingly against EFT (?)

3 AdS/CFT and BHIP

We will now turn our attention to the AdS/CFT picture of BHIP. This is a very interesting version of the information paradox, with the added intricacies of AdS/CFT. One can make sense of the usual properties of black holes in AdS in almost the same way, except that when needed there are some brane-y things. For instance, black hole evaporation in AdS is complicated and requires coupling the bulk gravitating region to a "bath". Most of what one does with islands and things requires doing so with branes in the geometry, but this is will be slightly put aside in the discussion. For instance, the temperature for an eternal AdS black brane is [5]

$$T = \frac{\hbar c \mathbf{d}}{4\pi z_H k_B} \ . \tag{3.1}$$

We will start by reviewing bulk fields and CFT operators outside and inside an AdS black hole, where the interpolation of operators in regions I and III (see fig. 7) are used to find operators in region II.

3.1 Raju-Papadodimas

The point of this subsection is to recap quickly an interesting result from Suvrat Raju and Kyriakos Papadodimas [5–7], regarding reconstruction of bulk fields in the interior of AdS black holes. We are, of course, missing a lot of details on the nature of partner operators. Ideally, we should have started with the splitting into coarse- and fine-grained Hilbert spaces, but the final result that we wanted is that the horizon interior is the same for all pure states. The infalling observer does not find anything special, and this is a contradiction to the fuzzballs proposal. We will once again be starting from solutions of the scalar field equation $(\Box - m^2)\Phi = 0$,

$$\xi_{\omega,k}(t, x, z) \ . \tag{3.2}$$

These can be expanded like

$$e^{-kx - i\omega t}\psi_{\omega,k}(z) \ , \tag{3.3}$$

where $\psi_{\omega,k}$ have a unique normalizable solution once we fix ω and k. We then identify three regions of interest in the spacetime – see fig. 7.

Fields in, say, region I of this geometry can be transformed into fields in region II by operating ϕ_I with a CPT-conjugation operator Θ_{CPT}:

$$\Theta^\dagger \phi_I \Theta = \phi_{III} \ , \tag{3.4}$$

where we have $\phi_I(r, t, \Omega)$ and $\phi_{III}(t, -t, \Omega)$. Decompose the respective Hamiltonians into right and left terms H_R and H_L. The basis of H_R eigenstates relate to those of H_L like

$$|i^*\rangle_L = \Theta^\dagger |i\rangle_R \ . \tag{3.5}$$

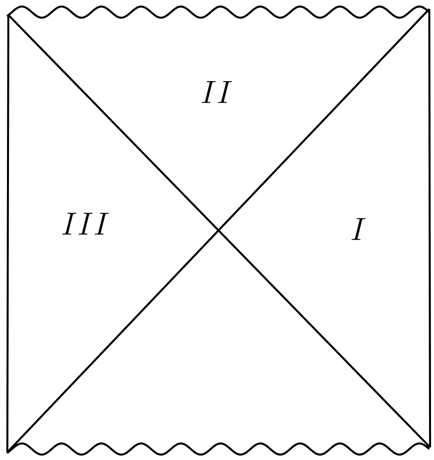

Figure 7. The three regions in the AdS black hole geometry. Bulk fields in region I are nicely make sense of from usual bulk reconstruction.

Now the usual quantization process applies, which is not very non-trivial. One has the usual expansion of a bulk field in region I in terms of creation and annihilation modes $a_{\omega,k}$ and $a^\dagger_{\omega,k}$:

$$\phi(t,x,z) = \int \frac{d\omega d^{D-1}k}{(2\pi)^D\sqrt{2\omega}} \left(a_{\omega,k}\tilde{\xi}_{\omega,k}(t,x,z) + \text{ Hermitian conjugates}\right) . \tag{3.6}$$

The modes $a_{\omega,k}$ and $a^\dagger_{\omega,k}$ satisfy the usual commutator rule. Similarly, one has modes in region III, which we will call $\tilde{a}_{\omega,k}$ and $\tilde{a}^\dagger_{\omega,k}$. Then, the pair of modes $a_{\omega,k}$ and $a^\dagger_{\omega,k}$, and $\tilde{a}_{\omega,k}$ and $\tilde{a}^\dagger_{\omega,k}$ together are used to construct region II.

Now, notice that in region I, we can define a CFT operator[1],

$$\phi^I_{CFT} = \int \frac{d\omega d^{D-1}k}{(2\pi)^D} \left(\mathcal{O}_{\omega,k}\xi_{\omega,k} + \mathcal{O}^\dagger_{\omega,k}\xi^*_{\omega,k}\right) . \tag{3.7}$$

One then defines operators $\tilde{\mathcal{O}}$, with the Fourier modes

$$\tilde{\mathcal{O}}_{\omega,k} = \int dtd^{D-1}x\, e^{-ikx+i\omega t}\tilde{\mathcal{O}}(t,x) . \tag{3.8}$$

With this, we can write an operator in region III like

$$\phi^{III}_{CFT} = \frac{d\omega d^{D-1}k}{(2\pi)^D} \left(\tilde{\mathcal{O}}_{\omega,k}\xi_{\omega,k} + \tilde{\mathcal{O}}^\dagger\xi^*_{\omega,k}\right) . \tag{3.9}$$

Here, the idea is that the partner operators are the Tilde-d ones in the thermofield double Hilbert space. Effectively, this allows us to write a nice description for the

[1]Throughout, for these integrals we take $\omega > 0$, although we do not explicitly mention it for convenience.

black hole interior. The point of all this is to similarly be able to write an expansion in region II. Take solutions to the Klein-Gordon equation,

$$g^{(1,2)}_{\omega,k} = e^{ikx - i\omega t}\chi^{(1,2)}(z) \ , \tag{3.10}$$

with the properties that will be discussed below. Then, we can write an operator in region II like

$$\phi^{II}_{CFT} = \int \frac{d\omega d^{D-1}k}{(2\pi)^D} \ \left(\mathcal{O}_{\omega,k} g^1_{\omega,k} + \tilde{\mathcal{O}}_{\omega,k} g^2_{\omega,k} + \text{ Hermitian conjugates}\right) \ . \tag{3.11}$$

One can explicitly compute the correlator

$$\langle \phi(x_1) \ldots \phi(x_n) \rangle_\Psi \tag{3.12}$$

in a pure CFT state $|\Psi\rangle$, telling us what an infalling observer would experience when going from region I to II. What Raju-Papadodimas tells us is that as Alice falls through the horizon (see fig. 8), she experiences natural physics, essentially augmenting **Postulate 4** of black hole complementarity. Clearly, this suggests that the argument of fuzzballs (or firewalls, although fuzzballs are more natural a description in comparison to firewalls) seems in contradiction with what one should and *does* expect of black hole horizons. There are some more arguments about what these correlators constitute and what the entire proposal on a complete scale would provide, in comparison to fuzzballs, but we will defer a discussion of those.

We will now turn to a second interesting aspect of AdS black holes, which has to do with coupling a bath to the bulk AdS and entanglement wedges.

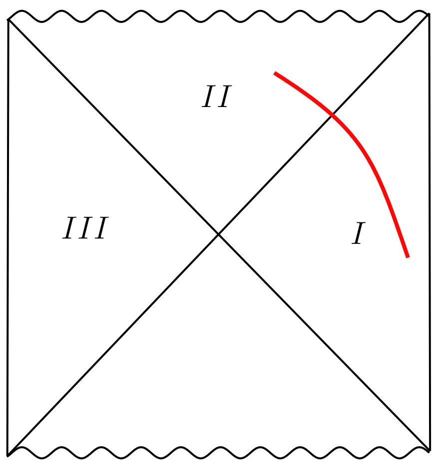

Figure 8. Infalling observer in the AdS black hole setup.

3.2 Entanglement Wedges

We will now discuss an intriguing aspect of entanglement wedges in AdS black hole spacetimes. Evaporation in AdS black hole spacetimes is weird and usually requires coupling to a *bath* [5, 8, 15]. Take the gravitating bulk AdS_D with the matter content a holographic CFT itself. This would have a dual CFT in $D-1$. We then couple this to a CFT_D in a flat background, and this is referred to as the AdS/CFT+bath system. For now, our interest would be in AdS/CFT with the following total action:

$$S(g^{(2)}_{\mu\nu}, \phi, \chi) = S_{grav}(g^{(2)}_{\mu\nu}, \phi) + S_{CFT}(g^{(2)}_{\mu\nu}, \chi) \ , \tag{3.13}$$

where ϕ is the dilaton field. The matter fields are taken to constitute a holographic CFT_2, and we locate the bath CFT as some CFT_2 (see upper fig. 9). One can then imagine the CFT_2 as living on the boundary of some AdS_3 theory, with the 2D dilaton theory description being on a Planck brane, as shown in lower fig. 9.

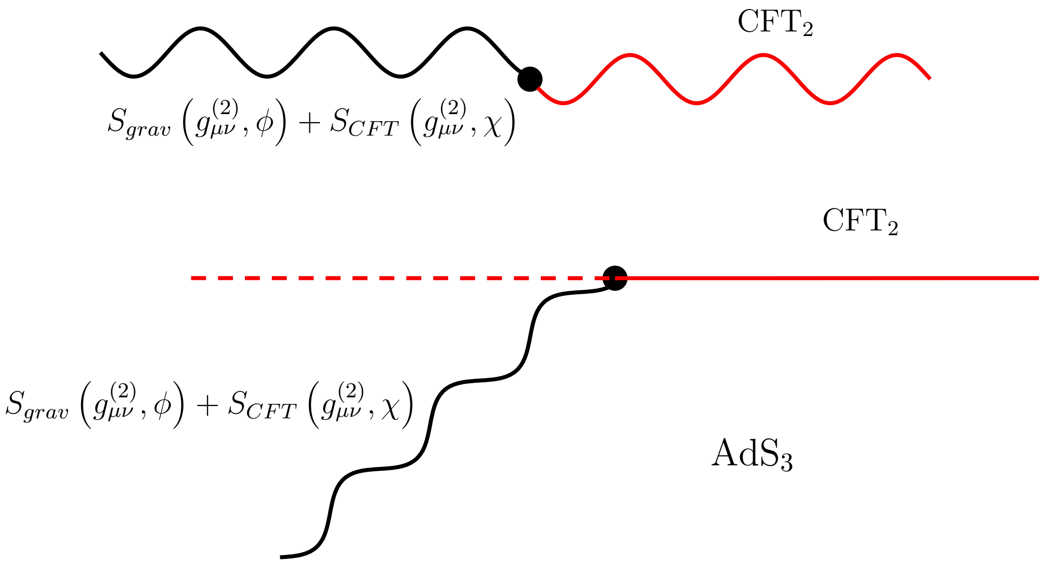

Figure 9. (Top) A CFT_2 bath (thick red wavey line) coupled to the 2D dilaton system. (Bottom) The 2D dilaton theory is on a Planck brane, with CFT_2 the dual to an AdS_3 bulk.

Let $\mathfrak{y}$ be a point in the 2D theory. The generalized entropy would be

$$S_{gen}(\mathfrak{y}) = \frac{\phi(\mathfrak{y})}{4G_N} + S_{bulk}(\mathbb{R}_{\mathfrak{y}}) \ , \tag{3.14}$$

where $\mathbb{R}$ is some interval to a weakly coupled region in the theory, and S_{bulk} measures the bulk von Neumann entropy of $\mathbb{R}_{\mathfrak{y}}$. For convenience, ignoring fluctuations of ϕ and the metric, we will write $S_{gen}(\mathfrak{y})$ as

$$S_{gen}(\mathfrak{y}) \sim \frac{\phi(\mathfrak{y})}{4G_N} + \frac{A(\mathcal{X}_{\mathfrak{y}})}{4G_N} \ . \tag{3.15}$$

The entanglement wedge corresponding to $\mathcal{X}_{\mathfrak{y}}$ can then be found out, which looks something like fig. 10.

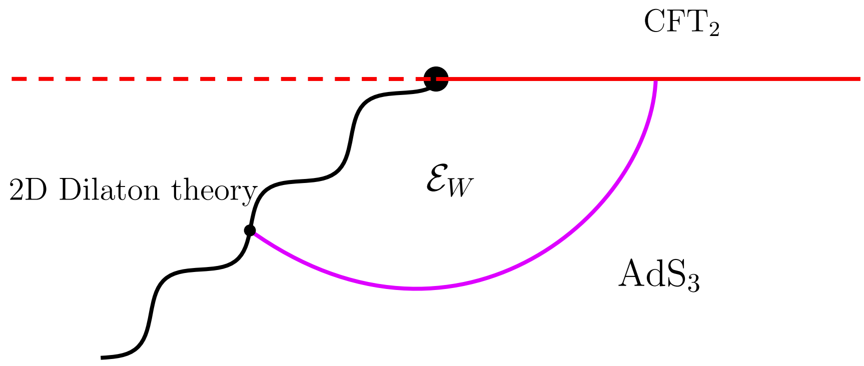

Figure 10. The entanglement wedge associated to $\mathcal{X}_\mathfrak{y}$.

What is interesting about this construction, among many things, is that this links to islands and entanglement wedges in an interesting fashion. Taking the HRT prescription into account with islands, one can formulate, in a quantum min-ext formalism, the entropy of some A in terms of the area of the boundary of an island + corrections:

$$S(A) = \min \operatorname{ext} \left(\frac{A(\partial I)}{4G_N} + S(A \cup I) \right) , \tag{3.16}$$

which also leads to an interesting Bousso-like bound for the generalized entropy of the entanglement wedge and a region A [8]:

$$S_{gen}(\mathcal{E}_W, A) < \frac{\text{Area of } (A)}{4G_N} . \tag{3.17}$$

3.3 An ER=EPR Paradox?

One remarkable thing about the eternal AdS black hole is that one can take the left and right boundary CFTs in the thermofield double state $|TFD\rangle$, which leads to an interesting paradox[2]. Start by noting that the TFD state is

$$|TFD\rangle = \mathcal{Z}_\beta^{-\frac{1}{2}} \exp -\frac{\beta E}{2} |E,\ E\rangle , \tag{3.18}$$

where $|E,\ E\rangle$ are eigenstates of the left and right Hamiltonians H_L and H_R. It is a very well-known thing that the state $|TFD\rangle$ is dual to an eternal AdS black hole. So now, one could say that operating on either CFT does not affect the other due to the nature of the TFD state. However, this leads to a paradox, which can be visualised by our usual protagonists Alice and Bob again. Let us say that Alice jumps in from the right wedge (an excitation on the right CFT), whereas Bob jumps in from the left wedge (an excitation on the left CFT). Since both fall into the black hole, where Alice and Bob meet is clearly affected by each other [9]. See fig. 11.

[2] I thank Aayush Verma for pointing out this paradox for inclusion in these notes.

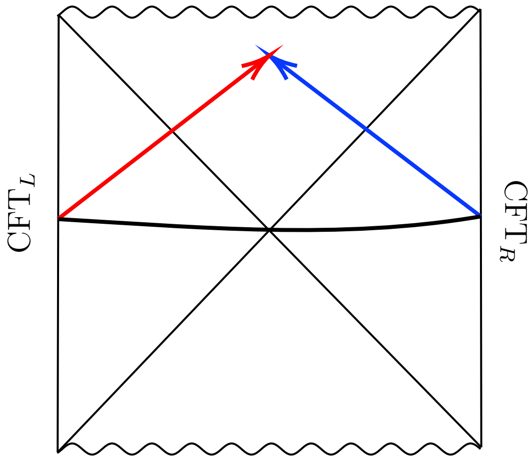

Figure 11. Alice (thick blue line) and Bob (thick red line) meeting behind the horizon.

The above thought experiment can be made more precise mathematically by identifying a state $|W\rangle$ and $|TFD\rangle$ as in Marolf and Wall's paper. The unitaries become

$$e^{iA} \quad \text{and} \quad e^{iB} \tag{3.19}$$

for Alice and Bob respectively. Then, the probabilities of meeting behind the black hole horizon is non-zero with Bob, that is,

$$\langle e^{-i(A+B)} \mathcal{P} e^{i(A+B)} \rangle_W \sim 1 \ , \tag{3.20}$$

whereas ~ 0 without Bob. A little more precision can be adopted by the use of superselection in the language [9], but we will not do so here.

3.4 Firewalls In Our Time

Let us reverse the locations of Alice and Bob, partly because Bob never does anything other than hover over a horizon while Alice does most of the work, and let us say that she now hates Bob. Alice could affect the boundary conditions on the left boundary CFT, so that it generates a firewall (with decay), which basically fries anyone it comes into contact with. One could ask if Bob experiences the firewall, which clearly is dependent on whether Alice shoots the shockwave in the first place [11].

If you imagine the two black holes to be very distant in one space without entanglement, there does not exist an ER bridge and all is good. However, suppose that both were prepared in an entangled state. Then, the ER bridge joining them is not traversable (in agreement with non-locality). However, what one *could* do, is to jump inside the black holes and meet behind the horizon in the entangled state, due to which communication would be possible. Of course, this seems impractical, but on a more cautious note, Alice may very well send in shockwaves, whereas Bob would jump in and get fried waiting for Alice.

It is, however, possible that there are traversable wormholes in this fashion of ER=EPR, as Jafferis, Gao and Wall showed [10], in which case there are some interesting observations linking to the above discussion. However, one still has to maintain that in the Alice-hates-Bob scenario, the systems are non-interacting, whereas in [10], this is not so. You can read the paper and find the consequences of the present situation for yourself.

4 Conclusion

In the grand scheme of theoretical high energy physics, there are few things more fascinating than the black hole information problem. In this discussion, we covered a few interesting things, like the flat space discussion of the BHIP in a pedagogical manner, particularly discussing some aspects of Hawking radiation, the Page curve, the generalized second law, complementarity and a monogamy paradox, with firewalls as a solution. We have not covered firewalls and fuzzballs with a lot of details, but as they stand, the discussion is sufficient for a motivation to the BHIP. In the AdS/CFT section, we discussed an argument on black hole interior reconstruction [7]. We then remarked on entanglement wedges and the coupling of a bath for the evaporation of an AdS black hole [8]. We then briefly discussed an ER=EPR paradox, and finally commented on traversability of wormholes and ER=EPR [9, 10]. While this discussion has not been very sophisticated with a lot of mathematics or lengthy elaboration, in the next Part, we will try to be a little more elaborate with some recent developments in the particular direction of Alheiri, Engelhardt, Marolf and Maxfield's work [16], along with Pennington's work on entanglement wedge reconstruction and BHIP [17].

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